VOLUME 77 SEPARATE No. 54

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

JANUARY, 1951

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

TRUSS DEFLECTIONS BY THE COORDINATE METHOD

By Kuang-Han Chu, Jun. ASCE

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Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

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Proceedings—Vol. 77 January, 1951 Separate No. 54

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

TRUSS DEFLECTIONS BY THE COORDINATE METHOD

By Kuang-Han Chu,[1] Jun. ASCE

Synopsis

The method presented here is an algebraic solution of the Williot-Mohr diagram. By suitable arrangement in tabular forms, and by adopting a set of simple sign conventions, the work is minimized in such a manner as to make this method preferable to the graphical method.

Introduction

There are many procedures for finding truss deflections. The virtual work method (the method of virtual velocity or dummy unit loading) may be used to find the movement of a single point in any direction; but to find the deflections at many points by this method is rather tedious. The elastic weight method (which has many variations and different names, such as the "influence line" method, the "bar chain" method, and the method of "angle changes") may be used to find the vertical components of deflections of the joints on a continuous chain of bars in a truss. Although the method can be modified to determine the horizontal components of movements by finding the influence lines due to a horizontal unit load, separate calculations must be made for the vertical and horizontal components. To find the true movements of all the joints of a truss, the elastic weight method may require as much time as the virtual work method. The Williot-Mohr method is the only simple direct procedure which determines the true (absolute) movements of all the truss joints. However, the method is a graphical one and its accuracy is limited. Sometimes one may have difficulty in selecting a proper scale and in orientating the diagram. The method presented here, which the writer has named the "coordinate method," is an algebraic solution of the Williot-Mohr diagram. Its application is simple and rapid. Vertical and horizontal components of deflections can be determined simultaneously for all the truss joints; and it can be carried to any degree of accuracy. The letter symbols used in this paper are defined where they first appear, in the text or by illustration, and are assembled in the Appendix for convenience of reference.

Note—Written comments are invited for publication; the last discussion should be submitted by July 1, 1951.

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PART I. PRINCIPLES OF THE METHOD AND WORKING PROCEDURE

Algebraic Solution of the Williot Diagram

Let δ be the change of length of any member, caused by stress, temperature change, error, or deliberate change in manufacture. If the change is caused by stress,

$$\delta = \frac{S\ L}{A\ E} \qquad \text{(1)}$$

in which S is the total stress in a member; L is the length of the member; A is the cross-sectional area; and E is the modulus of elasticity of its material. The symbol δ should have a plus sign for elongation and a minus sign for shortening. For any member PQ the deformation δ is designated as δ_{PQ}.

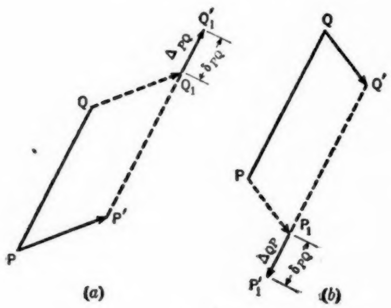


Fig. 1

Let Δ be the displacement of one end of a member, caused by the axial change in length δ of the member in its displaced position. For any member PQ, such a displacement is designated as Δ_{PQ} if the displaced position P′ of end P is known (see Fig. 1) and as Δ_{QP} if the displaced position Q′ of end Q is known. In Fig. 1, $P'Q_1$ and $P_1Q'_1$ are parallel and equal to PQ; the quantity $Q_1Q'_1$ represents Δ_{PQ}; and $P_1P'_1$ represents Δ_{QP}. The deformation δ_{PQ} shown in Fig. 1 is an elongation. It should be noted that $\Delta_{PQ} \neq \Delta_{QP}$ but that $\Delta_{PQ} = -\Delta_{QP}$. However, $\delta_{PQ} = \delta_{QP}$. In other words, Δ is a vector and δ is a scalar quantity.

The displacement Δ for a member is determined by the following conditions:

(*a*) Its magnitude is numerically equal to the deformation δ of the member;
(*b*) Its slope is the same as the original slope of the member; and
(*c*) Its direction depends not only on the sign of δ but also on the relative position between the end whose displaced position is known and the end whose displaced position is to be determined.

The displacement Δ can also be defined by its x-component and y-component The magnitude of each component is determined by conditions (*a*) and (*b*); and the sign of each component, by conditions (*b*) and (*c*). The x-component and y-component of any displacement Δ_{PQ} are designated as $(\Delta_{PQ})_x$ and $(\Delta_{PQ})_y$.

The following rules are suggested for the determination of the signs of Δ-components:

Rule I.—When the displaced position of the left end of a member is known, the components of Δ should bear signs depending on the sign of δ and the slope of the member as shown in Fig. 2(*a*), in which $+\ \delta$ means that the δ is positive (elongation) and $+\ m$ means that the slope is positive.

Rule II.—When the displaced position of the right end of a member is known, the Δ-components should bear signs depending on the sign of the deformation δ and the slope m of the member as shown in Fig. 2(*b*).

Rule III.—When the displaced position of the upper end of a vertical member is known, the y-component of Δ should bear a sign depending on the sign of the deformation δ of the member as shown in Fig. 2(*c*).

Rule IV.—When the displaced position of the lower end of a vertical member is known, the y-component of Δ should bear a sign depending on the sign of the corresponding deformation δ as shown in Fig. 2(*d*).

Generally, in a given truss, the new position of any joint can be determined, if, by means of a pair of bars, this joint is connected with two other joints whose displaced positions have been determined. (The new position of the joint is located at the intersection of the arcs drawn with the known new positions of the other two joints as centers and the new lengths of the connecting bars as radii.) Assume a bar to be fixed in direction and one of its ends to be fixed in position. The displaced position of this end is its original position. The displaced position of the other end is known when the deformation of the bar is known. Thus there are two points of known displaced positions to begin with.

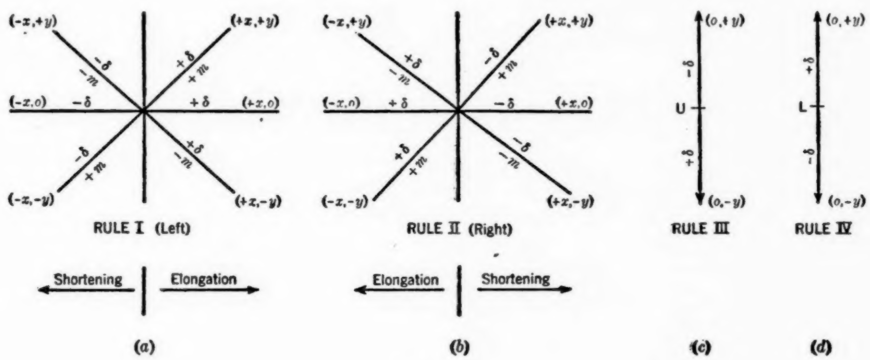


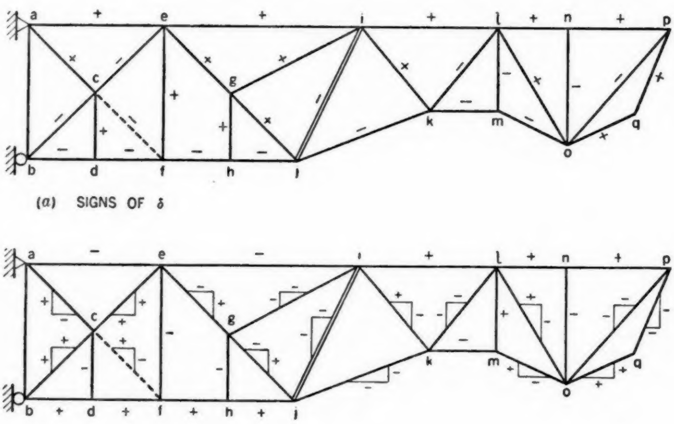
Fig. 2

Based on the rules given previously, the signs of Δ-components for all the members in a given truss can be determined easily. Fig. 3 illustrates all the possible cases. Although actually ab, Fig. 3, is the member of fixed direction and point a is the fixed point, it is assumed that member ij is the fixed member and point j, the fixed point. As point j is the left end of member ij, Rule I (with $-\delta_{ij}$ and a positive slope m of member ij) confirms that the x-component and the y-component of Δ_{ji} are both negative.

Consider first the members on the right side of member ij. With the displaced positions of joints i and j known, the displaced position of joint k can be determined as it is connected to joints i and j by the members ik and jk. Since points i and j are left ends of members ik and jk, respectively, Rule I is used for determining the signs of Δ-components. The same process is repeated for other joints and members.

For all the members on the right side of member ij except verticals and the member pq, the signs of Δ-components are governed by Rule I. The signs of y-components of Δ_{lm} and Δ_{on} are given by Rules III and IV, respectively, as ends l and o, determined by previous steps, are the upper and lower ends of member lm and member on, respectively. The signs of the components of Δ_{pq} are given by Rule II as the known end p is the right end.

Similarly, the signs of Δ-components for members on the left side of member ij can be determined. In this case Rule II applies to all members except verticals. Also, with the displaced positions of joints e and f known, the displaced position of joint c or joint d cannot be determined as neither is connected to points e and f by two members. Hence, a false member cf is added. The change of length of member cf is given by the relative movement of points c and f. The relative movement can be determined by virtual work. In using that method the only members that need to be considered are those forming the quadrilateral efdc. After the false member is inserted, the analyst proceeds as usual.



(a) SIGNS OF δ

(b) SIGNS OF Δ COMPONENTS

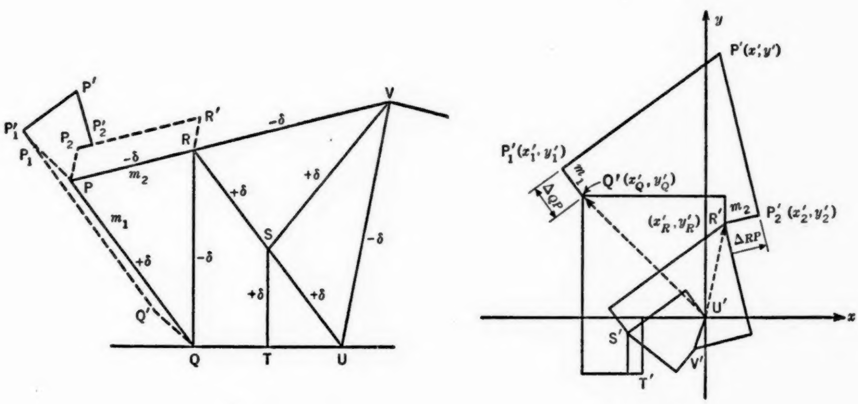
FIG. 3

There are seldom such exceptional members as pq and cf. In ordinary trusses, for all members except verticals, on the right side of the member assumed to be fixed in direction if it is inclined or vertical, or on the right side of the assumed fixed point if the assumed fixed member is horizontal, the signs of Δ-components are governed by Rule I, and those on the left side, by Rule II.

Let the joints on a given truss be A, B, C, etc., and let their displaced positions (corresponding points on the Williot diagram) be A′, B′, C′, etc., respectively. Fig. 4(a) shows a part of the truss; and Fig. 4(b), a part of the Williot diagram. Let joint U be assumed as fixed in position. The origin of the coordinate axes for the Williot diagram is taken at U′ which corresponds to point U, Fig. 4(a). For any point P, the coordinates are designated as x'_P and y'_P. These coordinates represent the components of the displacement from P to P′.

Let member UV be assumed as fixed in direction. Since one of its ends, U, is fixed in position, $x'_V = (\Delta_{UV})_x$ and $y'_V = (\Delta_{UV})_y$. With the displaced positions of points U and V (U′ and V′) known, the displaced positions of other joints can be located successively.

Consider a general case: A joint P is connected to joints Q and R by the bars PQ and PR (see Fig. 4). It is required to find the displaced position P′ of point P with the displaced positions (Q′ and R′) of Q and R known. Assume that joint P is disconnected. Consider the movement of end P of member QP. First joint P moves to point P_1; the displacement $\overrightarrow{PP_1}$ is equal to the displacement $\overrightarrow{QQ'}$ ($\overrightarrow{U'Q'}$ in the Williot diagram). Then point P_1 moves to point P'_1; the displacement $\overrightarrow{P_1P'_1}$ ($\overrightarrow{Q'P'_1}$ in Fig. 4(*b*)) is equal to Δ_{QP} which is parallel to member PQ. Then joint P'_1 moves along an arc with the new position of Q (point Q′) as center and the new length of PQ (line $Q'P'_1$) as radius. In a similar manner, end P of member RP moves first to point P_2, then to point P'_2, and then along an arc with point R′ as center and line $R'P'_2$ as radius.



(*a*) (*b*)

Fig. 4

The meeting of the two arcs determines the displaced position (P′) of joint P. Since the deformation of a member is very small as compared with its length, the arc of rotation is replaced by the tangent which is perpendicular to the radius vector parallel to the direction of the member.

Let the coordinates of P'_1 and P'_2 be x'_1, y'_1 and x'_2, y'_2, respectively. Then for joint P'_1,

$$x'_1 = x'_Q + (\Delta_{QP})_x \quad \text{(2a)}$$

$$y'_1 = y'_Q + (\Delta_{QP})_y \quad \text{(2b)}$$

and, for joint P'_2,

$$x'_2 = x'_R + (\Delta_{RP})_x \quad \text{(3a)}$$

$$y'_2 = y'_R + (\Delta_{RP})_y \quad \text{(3b)}$$

Eqs. 2 and 3 are to be applied to general cases and all terms in the formulas should have proper signs. Let the slopes of members PQ and PR be m_1 and m_2, respectively (slopes according to usual sign convention). The equations of the perpendiculars are

$$y' - y'_1 = -\frac{1}{m_1}(x' - x'_1) \qquad \text{(4a)}$$

and

$$y' - y'_2 = -\frac{1}{m_2}(x' - x'_2) \qquad \text{(4b)}$$

Solve Eqs. 4 simultaneously. The coordinates of P′ (that is, x', y', or x'_P, y'_P), representing the components of the displacement of P, are

$$x' = \frac{(y'_1 - y'_2) + \left(\dfrac{x'_1}{m_1} - \dfrac{x'_2}{m_2}\right)}{\dfrac{1}{m_1} - \dfrac{1}{m_2}} \quad \left(\text{if } \frac{1}{m_1} = \infty, \text{ then } x' = x'_1\right) \qquad \text{(5a)}$$

and

$$y' = \frac{(x'_1 - x'_2) + (m_1 y'_1 - m_2 y'_2)}{m_1 - m_2} \quad (\text{if } m_1 = \infty, \text{ then } y' = y'_1) \qquad \text{(5b)}$$

Eqs. 5 are applicable to all possible cases. The special cases $\frac{1}{m_1} = \infty$ and $m_1 = \infty$ apply to horizontal and vertical members, respectively.

For the practical solution of the problem, the procedure is as follows:

1. Compute and record on each member in the truss diagram the values of δ (+ for elongation and − for shortening).
2. Mark the member that is actually (or assumed to be) fixed in direction and mark one of its ends that is actually (or assumed to be) fixed in position. Write the sign conventions for Δ-components.
3. Draw another truss diagram and record on each member the following:
   (*a*) The slope m of the member and its reciprocal value, $1/m$ (plus or minus according to the usual convention.)
   (*b*) Vertical and horizontal components of Δ, with signs according to step 2.
4. Write the formulas to be used if necessary.
5. Applying Eqs. 2, 3, and 5, compile a table of the form described in Part II as Table 1.

Algebraic Solution of the Mohr Diagram and Actual Displacements of Truss Joints

Five general principles for the solution of the Mohr diagram are as follows:

Principle 1.—Let F be an actual fixed point and let its corresponding points on the Williot and Mohr diagrams be F′ and F″, respectively. Then F′ and F″ are a common point.

Let any truss point P and its corresponding points on the Williot and Mohr diagrams be P′ and P″, respectively. Let the coordinates of P′ and P″ with

respect to F′ or F″ be x'_{FP}, y'_{FP} and x''_{FP}, y''_{FP}, respectively. If the Williot diagram is started from an assumed fixed point U instead of the actual fixed point F and the coordinates of P′ and P″ with respect to U′ are x'_P, y'_P and x''_P, y''_P, respectively, then the following relations can be established:

$$x'_{FP} = x'_P - x'_F;\ y'_{FP} = y'_P - y'_F \qquad (6a)$$

and

$$x''_P = x''_{FP} + x''_F;\ y''_P = y''_{FP} + y''_F \qquad (6b)$$

In Eqs. 6, $x'_F = x''_F$ and $y'_F = y''_F$. Also, if the Williot diagram starts from the actual fixed point F, then $x'_F = x''_F = y'_F = y''_F = 0$.

Principle 2.—Let point M be constrained to move in a fixed direction. The direction of motion may be given, such as the case of a roller; or it may be determined by known conditions, such as the case of the crown hinge of a three-hinged arch. Let M′ and M″ be its corresponding points on the Williot and Mohr diagrams, respectively. Then point M″ can be located when the positions of points M′ and F′ are known.

Consider first the case in which the direction of motion of point M is given. According to the principle of the Mohr diagram, the actual movement of point M is represented by the displacement from M″ to M′ (that is, $\overrightarrow{M''M'}$) which is along the known direction. The displacement $\overrightarrow{M''M'}$ is obtained by the vectorial relation $\overrightarrow{M''M'} = \overrightarrow{F'M'} - \overrightarrow{F'M''}$ in which $\overrightarrow{F'M'}$ is the displacement of point M with respect to point F as given by the Williot diagram and $\overrightarrow{F'M''}$ is the rotational displacement of point M about point F.

Hence point M″ should lie on the line through point M′ and parallel to the known direction and it should lie on the line (arc) through point F′ and perpendicular to line FM. Let the slope of the line of known direction of motion be m_o and the slope of the line normal to that direction be m'_o, then

$$m'_o = -\frac{1}{m_o} \qquad (7)$$

The coordinates of point M″ can be found by applying Eqs. 5 with the coordinates of point M′ as x'_1, y'_1, the coordinates of point F′ as x'_2, y'_2, the slope m'_o as m_1, and the slope of line FM as m_2.

Consider next the case in which the direction of motion is determined by known conditions. For example, a three-hinged arch is shown in Fig. 5. The usual graphical method of finding the movement of the crown hinge is as follows: Beginning at a common point a′,e′ (because points a and e are given fixed points), two Williot diagrams are drawn for two halves of the arch—one for the left half, assuming that line ai remains vertical, and the other for the right half, assuming that line eg remains vertical. The positions of points c and d are given as points c′ and d′, respectively, in the Williot diagram. However, c and d represent a common point; hence c must lie on the line (arc) through c′ and perpendicular to ac and d on the line (arc) through d′ and perpendicular to ed. These lines intersect at the common point c‴,d‴ and $\overrightarrow{a'c'''}$ or $\overrightarrow{e'd'''}$ gives the true movement of the common point c,d. Point c″

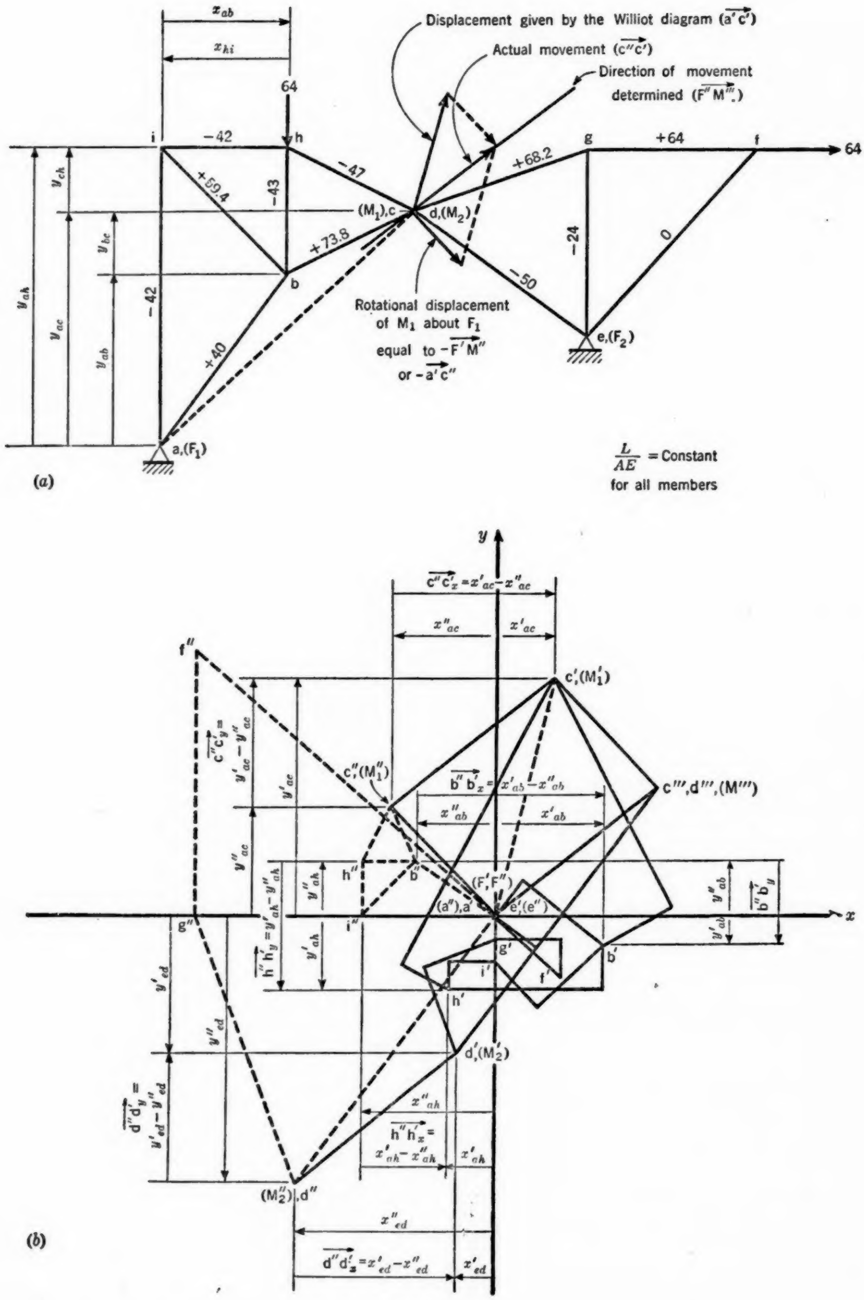

Displacement given by the Williot diagram (a'c')
Actual movement (c''c')
Direction of movement determined (F''M''')
64
i
h
g
f
64
−42
+64
+68.2
−47
+59.4
−43
(M1),c
d,(M2)
+73.8
b
−24
0
−50
−42
+40
Rotational displacement of M1 about F1 equal to −F'M'' or −a'c''
e,(F2)
a,(F1)
L/AE = Constant for all members
(a)
y
x
c''c'x = x'ac − x''ac
c',(M'1)
c'',(M''1)
c''',d''',(M''')
b''b'x = x'ab − x''ab
(F',F'')
(a''),a'
e',(e'')
g''
g'
i'
f'
b'
h'
d',(M'2)
(M''2),d''
h''h'x = x'ah − x''ah
d''d'x = x'ed − x''ed
(b)

FIG. 5

on the Mohr diagram (corresponding to point c′ on the Williot diagram) must lie on a line through a′ and perpendicular to ac, and on a line through c′ parallel to the known direction of motion as given by $\overrightarrow{a'c'''}$. Similarly, point d″ (corresponding to point d′) can be located.

The algebraic solution of the foregoing problem can be worked out according to the same principle as the graphical method. For instance, with the coordinates of points c′ and d′ known and the slopes of lines ac and cd given, the coordinates of the common point c‴,d‴ can be found by applying Eqs. 5. Referring to the common origin a′,e′ as F′ and the common point c‴,d‴ as M‴, then $\overrightarrow{F'M'''}$ represents the true movement of the crown hinge. The slope of the normal to the displacement $\overrightarrow{F'M'''}$ can be determined by its components $\overrightarrow{F'M'''_x}$ and $\overrightarrow{F'M'''_y}$ (signs of components referring to F′ as origin), as follows:

$$m'_o = -\frac{\overrightarrow{F'M'''_x}}{\overrightarrow{F'M'''_y}} \qquad (8)$$

Following the same procedure as established for the previous case with a point of given direction of motion, the coordinates of points c″ and d″ can be determined. For instance, the coordinates of point c″ can be found by applying Eqs. 5 with the coordinates of points c′ and a′ as x'_1, y'_1 and x'_2, y'_2, respectively, and the slope m'_0 as m_1 and the slope of line ac as m_2.

Principle 3.—The coordinates x''_{FP}, y''_{FP} of any point P″ on the Mohr diagram with respect to point F″ (coinciding with point F′) can be found when the coordinates x''_{FM}, y''_{FM} are known. The coordinates x''_{FM} and y''_{FM} represent, respectively, $\overrightarrow{F'M''_x}$ and $\overrightarrow{F'M''_y}$, the x-component and the y-component of the displacement $\overrightarrow{F'M''}$. Let the coordinates of any point P in the truss diagram with respect to the vertical and horizontal axes through point F be x_{FP} and y_{FP}. For point M, the coordinates are x_{FM} and y_{FM}.

Calculate the "angle of rotation" θ by the following formula:

$$\theta = \frac{\overrightarrow{F'M''_y}}{x_{FM}} = \frac{y''_{FM}}{x_{FM}} = -\frac{\overrightarrow{F'M''_x}}{y_{FM}} = -\frac{x''_{FM}}{y_{FM}} \qquad (9)$$

(In Eq. 9, angle θ is considered as positive when the rotation is counterclockwise.) Consider a radius vector FM with positive slope (that is, x_{FM} and y_{FM} are both positive) rotating about its left end F counterclockwisely through an angle θ. The displacement of the right end as represented by $\overrightarrow{F'M''}$ (an arc of rotation considered to be perpendicular to the radius vector) will be upward (positive $\overrightarrow{F'M''_y}$) and toward the left (negative $\overrightarrow{F'M''_x}$). Hence the sign for the term $\overrightarrow{F'M''_y}/x_{FM}$ is positive and the sign for the term $\overrightarrow{F'M''_x}/y_{FM}$ is negative.

The coordinates of truss points with respect to F can be written directly by consulting the truss diagram. However, sometimes it may be convenient to calculate these coordinates successively from point to point. If Q and P are two consecutive points, then

$$x_{FP} = x_{FQ} + x_{QP};\ y_{FP} = y_{FQ} + y_{QP} \qquad (10)$$

in which x_{QP} and y_{QP} are projections of the length of the member QP. They have algebraic signs with respect to the vertical and horizontal axes through Q.

The rotational displacement for any point about F is equal to the radius vector from F to that point times the angle of rotation. Thus, for any point P,

$$x''_{FP} = \overrightarrow{F'P''_x} = -\theta\, y_{FP} \qquad (11a)$$

and

$$y''_{FP} = \overrightarrow{F'P''_y} = +\theta\, x_{FP} \qquad (11b)$$

Eqs. 11 are based on the same principle as Eq. 9.

Knowing the values x''_{FP} and y''_{FP}, the coordinates x''_P y''_P of the point P″ with respect to the assumed fixed point U′ can be determined by Eq. 6*b*.

Principle 4.—The actual movement of a point is given by the vector from the corresponding point on the Mohr diagram to the corresponding point on the Williot diagram. It can also be obtained by subtracting the rotational displacement from the displacement determined by the Williot diagram with respect to point F. Hence, for any point P, the components of the actual movement are $\overrightarrow{P''P'_x}$ and $\overrightarrow{P''P'_y}$ as given by the following formulas:

$$\overrightarrow{P''P'_x} = x'_P - x''_P;\ \overrightarrow{P''P'_y} = y'_P - y''_P \qquad (12a)$$

or

$$\overrightarrow{P''P'_x} = x'_{FP} - x''_{FP};\ \overrightarrow{P''P'_y} = y'_{FP} - y''_{FP} \qquad (12b)$$

Principle 5.—In certain special cases, the algebraic solution of the Mohr correction diagram can be much simplified. Let L_i and U_i be, respectively, the lower and upper panel points of an ordinary truss, in which $i = 0, 1, 2, 3, \ldots, n$. If L_o at the left end is a fixed point and L_n at the right end is movable in horizontal direction only, then (see Fig. 6)

$$\theta' = \frac{y'_{Ln} - y'_{Lo}}{\overline{L_o L_n}} \qquad (13)$$

If, in addition to the foregoing conditions, the lower chord is horizontal and the upper chord U_hU_i in the panel *h-i* (panel length $= \overline{L_hL_i}$ and $h = i - 1$) has a slope equal to m_{ci} (the end post also being considered as an upper chord), then (see Fig. 6)

$$x''_{Li} = x'_{Lo} \qquad (14a)$$

$$x''_{Ui} = x'_{Lo} - \theta' \sum_{i=1}^{i=i} (\overline{U_hU_i})_y = x'_{Lo} - \theta' \sum_{i=1}^{i=i} \overline{L_hL_i}\, m_{ci} =$$

$$x''_{Lh} - \theta' (\overline{U_hU_i})_y \qquad (14b)$$

$$y''_{Li} = y'_{Lo} + \theta' \sum_{i=1}^{i=i} \overline{L_hL_i} = y''_{Lh} + \theta' \overline{L_hL_i} \qquad (14c)$$

and

$$y''_{Ui} = y''_{Li} \qquad (14d)$$

in which $(\overline{U_hU_i})_y$ is the *y*-projection of the length $\overline{U_hU_i}$ bearing algebraic signs with respect to the vertical and horizontal axes through U_h. If, in addition to

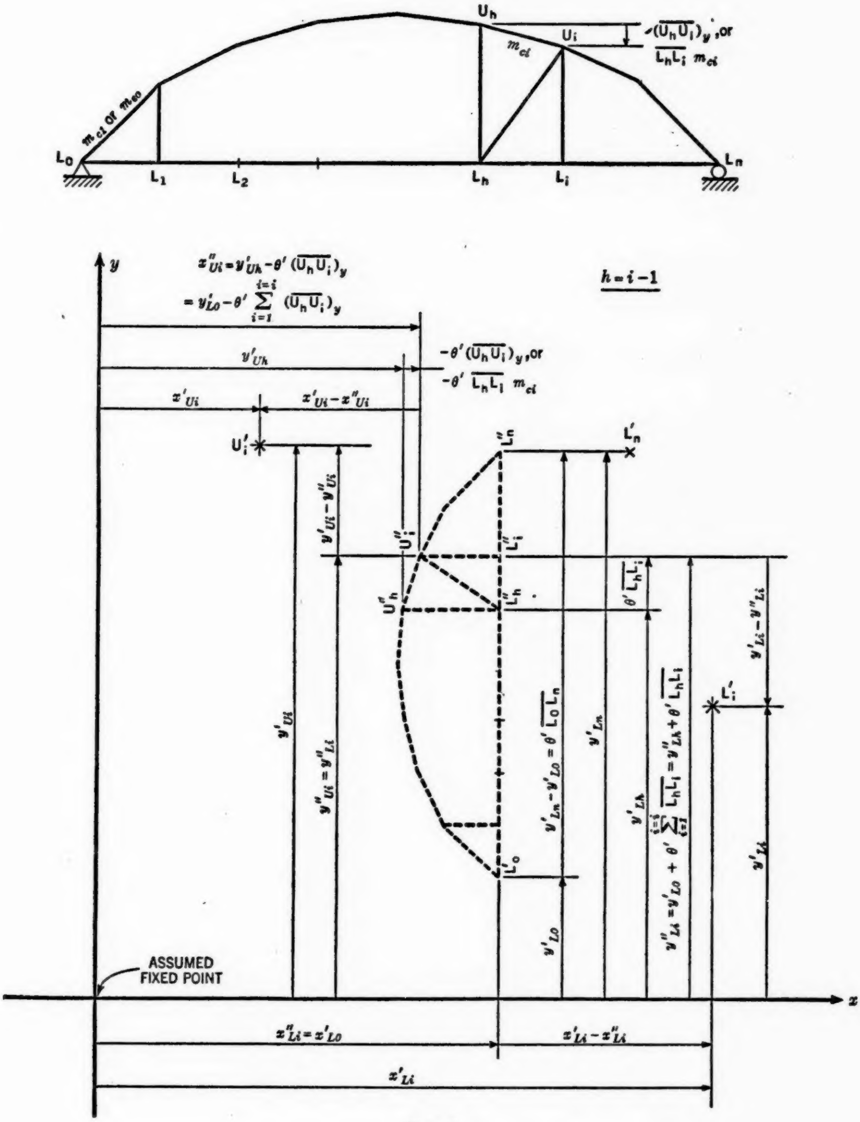


FIG. 6

the foregoing conditions, the truss is of equal panel length, then instead of Eqs. 13, 14a, and 14b, respectively, the following equations may be used:

$$\theta'' = \frac{y'_{Ln} - y'_{Lo}}{n} \quad \text{.....................} (15a)$$

$$x''_{Ui} = x'_{Li} - \theta'' \sum_{i=1}^{i=i} m_{ci} = x''_{Lh} - m_{ci}\, \theta'' \quad \text{...........} (15b)$$

and

$$y''_{Li} = x'_{Lo} + i\,\theta'' \qquad \text{(15c)}$$

If, in addition to the foregoing conditions, the truss has a horizontal upper chord, then, instead of Eq. 15*b*,

$$x''_{Ui} = x'_{Lo} - m_{eo}\,\theta'' \qquad \text{(16)}$$

in which m_{eo} is the slope of the end post at point L_o. The actual movements for all the aforementioned cases are (corresponding to Eqs. 12)

$$\left.\begin{aligned} x\text{-movement of } L_i &= (\overrightarrow{L''_iL'_i})_x = x'_{Li} - x''_{Li} \\ y\text{-movement of } L_i &= (\overrightarrow{L''_iL'_i})_y = y'_{Li} - y''_{Li} \end{aligned}\right\} \qquad \text{(17a)}$$

and

$$\left.\begin{aligned} x\text{-movement of } U_i &= (\overrightarrow{U''_iU'_i})_x = x'_{Ui} - x''_{Ui} \\ y\text{-movement of } U_i &= (\overrightarrow{U''_iU'_i})_y = y'_{Ui} - y''_{Ui} \end{aligned}\right\} \qquad \text{(17b)}$$

PART II. ILLUSTRATIONS AND TABULAR FORMS

To illustrate the principles stated in Part I, an example and several tabular forms are presented herein. The general method of solution will be illustrated first and some special cases will be discussed subsequently.

An Example for the General Method of Solution

Consider the three-hinged arch shown in Fig. 5(*a*). It is required to find the deflections of the joints under the loading shown in the figure by the coordinate method.

Step 1. Deformations and Dimensions.—The δ-components, the load, and the dimensions of the truss are shown in Fig. 7. For simplicity, the $\dfrac{L}{A\,E}$-values

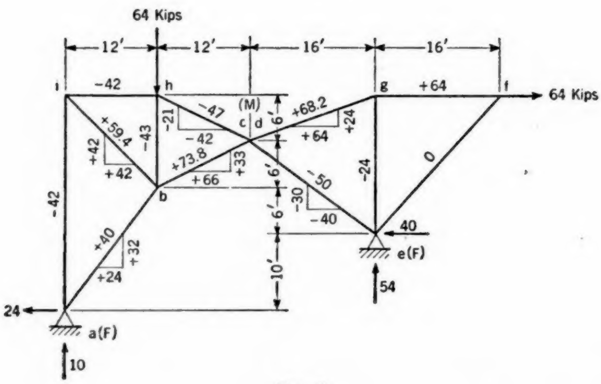


Fig. 7

are assumed to be equal to a constant K for all members. Hence, the δ-components are coefficients of K and are equal to the stress components.

Step 2. Sign Conventions.—The two halves of the arch on the left and the right sides of the crown joint are considered separately. In these two

halves, a and e are considered as the respective fixed points and ai and eg as the respective bars fixed in direction (see Fig. 8). Note the different sign conventions for Δ-components used for the left and the right sides of a member assumed as fixed in direction. (In this example, there is no such exception as the member pq in Fig. 3.)

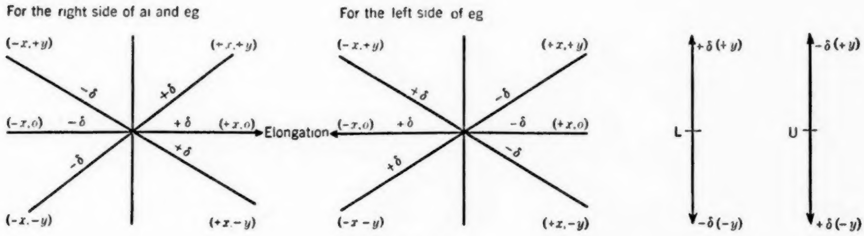


Fig. 8

Step 3. Slope and Displacement.—In Fig. 9, observe the change from the signs of δ-components to the signs of Δ-components.

Step 4. Formulas.—Eqs. 3 and 5 are used for the algebraic solution of the Williot diagram. For example, referring to Eq. 2*a*, $x'_{1,P} = x'_Q + (\Delta_{QP})_x$, and similarly for Eqs. 2*b*, 3, and 5.

Step 5.—The algebraic solution of the Williot diagram is demonstrated in Table 1.

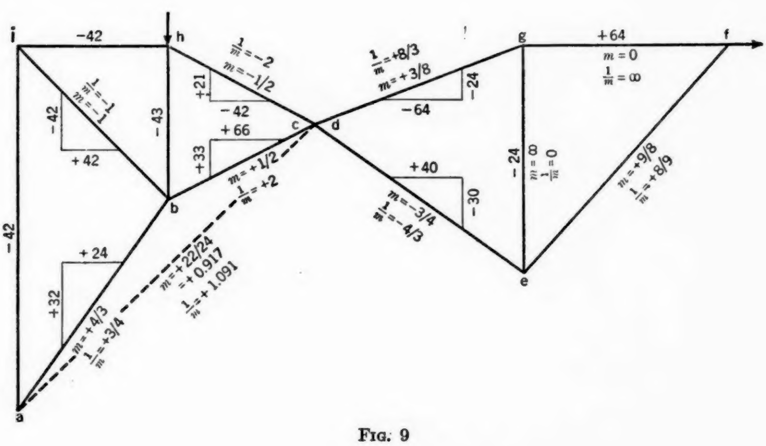


Fig. 9

The actual fixed points and the points that move in directions fixed by known conditions are designated F and M in subscripts, respectively. As point a is fixed in position, the coordinates of a′, item 1, are 0 as given in Cols. 4 and 8. Since line ai is assumed to be fixed in direction, the coordinates of i′, item 2, are the x-component and the y-component of Δ_{ai}. These components are also entered in Cols. 4 and 8, respectively.

With the coordinates of a′ and i′ known, point b′ can be located since b is connected to points a and i by the bars ab and ib, respectively. Referring to Col. 4, the x-value of i′ plus the x-component of Δ_{ib} in item 3 gives x'_1 in item 4. (This value is designated x'_1 in applying Eqs. 5. Col. 4, Table 1, is headed x

TABLE 1.—Coordinates of the Points on the Williot Diagram

Item	Joint[a] (1)	Member (2)	x (3)	x (4)	$\frac{1}{m}$ (5)	$\frac{x}{m}$ (6)	y (7)	y (8)	m (9)	$m\,y$ (10)
1	a'_F			[0]				[0]		
2	i′			[0]				[−42]		
3		ib		+42				−42		
4				+42	−1	−42		−84	−1	+84
5		ab		+24	+3/4	+18		+32	+4/3	+42.67
6				+18	−7/4	−60		−116	−7/3	+41.33
7						−116[b]				+18.00[c]
8	b′			[+100.57] ×	−7/4 =	−176		[−25.43] ×	−7/3 =	+59.33
9		bh		0				−43.00		
10				+100.57	0			−68.43	∞	
11		ih		−42.00	∞			0		
12	h′			[−42.00][d]				[−68.43][e]		
13		hc		−42				+21		
14				−84	−2	+168.00		−47.43	−1/2	+23.72
15	b′		+100.57				−25.43			
16		bc	+66				+33			
17			+166.57	+166.57	+2	+333.14	+7.57	+7.57	+1/2	+3.79
18				−250.57	−4	−165.14		−55.00	−1	+19.93
19						−55.00[b]				−250.57[c]
20	c'_M			[+55.04] ×	−4 =	−220.14		[+230.64] ×	−1 =	−230.64
21	e'_F			[0]				[0]		
22	g′			[0]				[−24]		
23		gd		−64				−24		
24				−64	+8/3	−170.67		−48	+3/8	−18.0
25		ed		+40	−4/3	−53.33		−30	−3/4	+22.5
26				−104	+4	−117.34		−18	+9/8	−40.5
27						−18.00[b]				−104.0[c]
28	d'_M			[−33.84] ×	+4 =	−135.34		[−128.44] ×	+9/8 =	−144.5
29	e′			0				0		
30		ef		0	+8/9			0	+9/8	0
31	g′		0				−24			
32		gf	+64				0			
33			+64	+64	∞		−24	−24	0	0
34				−64				+24	+9/8	0
35										−64[c]
36	f′			[+64]				[−56.89] ×	+9/8 =	−64

[a] Subscript F denotes fixed joints and subscript M denotes points that move in directions fixed by known conditions. [b] From Col. 8. [c] From Col. 4. [d] Since $1/m_2 = \infty$, $x' = x'_2$ and this item is the same as the item above it. [e] Since $m_1 = \infty$, $y' = y'_1$ and this item is the same as the second item above it.

without any subscript because the column is not restricted to values of x'_1. Other notations like x'_2, m_1, m_2, y'_1, and y'_2, used in this section have meanings similar to x'_1.) This value (item 4, Col. 4), multiplied by $1/m_1$ for line ib in Col. 5, gives x'_1/m_1 in Col. 6. The x-value of point a′, plus the x-component of

Δ_{ab} gives x'_2 in item 5. The process of addition is omitted since the x-value of point a′ is zero. The x'_2-value in item 5 is then multiplied by $1/m_2$ for line ab in item 5 to produce x'_2/m_2 in item 5, Col. 6. The values $x'_1 - x'_2$, $\frac{1}{m_1} - \frac{1}{m_2}$, and $\frac{x'_1}{m_1} - \frac{x'_2}{m_2}$ are recorded as item 6 in Cols. 4, 5, and 6, respectively. The same procedure is used to obtain $y'_1 - y'_2$, $m_1 - m_2$, and $m_1 y'_1 - m_2 y'_2$ in item 6, Cols. 8, 9, and 10, respectively. The value in item 6, Col. 4, is then carried to item 7, Col. 10, and it, plus the value of $m\,y'$ in item 6, Col. 10, gives the value entered in item 8, Col. 10. This value is $(x'_1 - x'_2) + (m_1y'_1 - m_2y'_2)$ and it is divided by $m_1 - m_2$, the value in item 6, Col. 9, to produce the y-value for point b′ in line 8, Col. 8. Similarly, the x-value for point b′ is determined and entered in item 8, Col. 4.

With the coordinates x and y of points b′ and i′ known, point h′ can be located. The x-value for point b′, plus the x-component of Δ_{bh} in item 9, Col. 4, gives x'_1 in item 10, Col. 4. The values in Cols. 5 and 9, item 10, need no explanation. The value x'_2 in item 11, Col. 4, is known to be the x-component of Δ_{ih}. However, it is to be noted that the ratio $1/m_2$ for the member ih is ∞. Therefore, x for point h′ in item 12, Col. 4, is the same as x'_2 in item 11. Similarly, the y-value of point h′ is determined.

With x and y of points h′ and b′ known, point c′ can be located. Again, referring to Col. 4, the x'_1-value in item 14 is obtained by the addition of the x-value of point h′ in item 12 and the x-component of Δ_{hc} in item 13. The x'_2-value in item 17, Col. 3, is obtained by the addition of the x'_1-value of point b′ (from item 8, Col. 4) in item 15, Col. 3, and the x-component of Δ_{bc} in item 16, Col. 3. This step is shown in Col. 3 instead of Col. 4 to avoid confusing this step with the next. Then the x'_2-value in item 17, Col. 3, is carried to item 17, Col. 4, and it is subtracted from x'_1 in item 14, Col. 4, to give $x'_1 - x'_2$ in item 18, Col. 4. Col. 7 for y serves the same purpose as Col. 3 for x. Follow the usual procedure to obtain the x-value and the y-value of point c′.

Starting anew from point e′, locate points g′, d′, and f′ in the same manner as indicated previously. In Col. 4, note that, although the value in item 36 is equal to x'_2 in item 33 for m_2 is ∞, the x'_2-value in item 33 is subtracted from x'_1 in item 30 to obtain the value $x'_1 - x'_2$ in item 34. The value $x'_1 - x'_2$ is necessary for computing y' in item 36, Col. 8. However, in the case of a joint such as h, which is located by one vertical bar and one horizontal bar, neither $x'_1 - x'_2$ nor $y'_1 - y'_2$ is necessary (see items 10 to 12).

Step 6. Algebraic Solution of the Mohr Diagram.—The computations for the rotational correction of the Mohr diagram are carried out in Table 2, which is based on Principles 1, 2, and 3 (under the heading, "Algebraic Solution of the Mohr Diagram and Actual Displacements of Truss Joints") in Part I. Its form is almost the same as that of Table 1, although Cols. 3 and 7, being unnecessary in this example, have been omitted. The coordinates of point c‴ in Cols. 4 and 8, item 5, are determined according to Principle 2 stated in Part I. Item 6 can be omitted for this example because the origin of the Williot diagram is an actual fixed point. However, when it is not actually fixed, the x-component and the y-component of $\overline{\text{F}'\text{M}'''}$ in Cols. 4 and 8, item 7, should be given by the

differences between items 5 and 6, Col. 4, and items 5 and 6, Col. 8, respectively. The m-value in item 8, Col. 9, is $m'_o = -\dfrac{\overrightarrow{\mathrm{F'M'''}}_x}{\overrightarrow{\mathrm{F'M'''}}_y} = -\dfrac{\text{Col. 4, item 7}}{\text{Col. 8, item 7}}$. The ratio $1/m$ in item 8, Col. 5, is the reciprocal of item 8, Col. 9. The coordinates of point c″ in Cols. 4 and 8, item 13, and of point d″ in Cols. 4 and 8, item 20, are determined according to Principle 2 stated in Part I. Items 14 and 15, and items 21 and 22, serve the same purpose as items 6 and 7, respectively. Items 23 to 26, Table 2, are for the calculation of the angle of rotation θ. The

TABLE 2.—ALGEBRAIC SOLUTION OF THE MOHR DIAGRAM; COMPUTATION OF CORRECTIONS

Item	Joint (1)	Member (2)	x (3)(4)	$1/m$ (5)	x/m (6)	y (7)(8)	m (9)	$m\,y$ (10)
1	d′	ed	−33.84 ×	−4/3 =	+45.12	−128.44 ×	−3/4 =	+96.33
2	c′	ac	+55.04 ×	+12/11 =	+60.04	+230.64 ×	+11/12 =	+211.42
3			−88.88	−80/33	−14.92	−359.08	−5/3	−115.09
4					−359.08[a]			−88.88[b]
5	c'''_M		[+154.28]		−374.00	[+122.38]		−203.97
6	a′		0			0		
7		a′c‴, $\overrightarrow{\mathrm{F'M'''}}$	[+154.28]			[+122.38]		
8		a′c‴ or e′d‴		[−0.7932][c]			[−1.2607][c]	
9	c′	a′c‴	+55.04	−0.7932	−43.66	+230.64	−1.2607	−290.77
10	a′	ac	0	+1.0909	0	0	+0.9167	0
11			+55.04	−1.8841	−43.66	+230.64	−2.1774	−290.77
12					+230.64[a]			+55.04[b]
13	c''_M		[−99.24] ×	−1.8841 =	+186.98	[+108.26] ×	−2.1774 =	−235.73
14	a'_F		0			0		
15		a′c″, $\overrightarrow{\mathrm{F'M''}}$	[−99.24]			[+108.26]		
16	d′	e′d‴	−33.84	−0.7932	+27.84	−128.44	−1.2607	+161.92
17	e′	ed	0	−1.3333	0	0	−0.7500	0
18			−33.84	+0.5401	+26.84	−128.44	−0.5107	+161.92
19					−128.44[a]			−33.84[b]
20	d''_M		[−188.11] ×	+0.5401 =	−101.60	[−250.79] ×	−0.5107 =	+128.08
21	e'_F		0			0		
22		e′d″, $\overrightarrow{\mathrm{F'M''}}$	[−188.11]			[−250.79]		
23	Length	ac	+24			+22		
24	$\overline{\mathrm{FM}}$	ed	−16			+12		
25	$\theta = \dfrac{\overrightarrow{\mathrm{F'M''}}_y}{x_{FM}} = -\dfrac{\overrightarrow{\mathrm{F'M''}}_x}{y_{FM}}$		$\theta_1 = \dfrac{\overrightarrow{\mathrm{a'c''}}_y}{x_{ac}} = +\dfrac{108.26}{+24} = -\dfrac{\overrightarrow{\mathrm{a'c''}}_x}{y_{ac}} = -\dfrac{(-99.24)}{+22} = +4.5108$					
26			$\theta_2 = \dfrac{\overrightarrow{\mathrm{e'd''}}_y}{x_{ed}} = -\dfrac{250.79}{-16} = -\dfrac{\overrightarrow{\mathrm{e'd''}}_x}{y_{ed}} = -\dfrac{(-188.11)}{+12} = +15.675$					

[a] From Col. 8. [b] From Col. 4. [c] Based on Eq. 8.

coordinates of the point of known direction of motion, M, with respect to the fixed point F (that is, the projections of line FM) should have proper signs. For example, from the truss diagram of this example, point d_M is on the left side of the point e_F; hence, x_{ed} in item 24, Col. 4, is negative.

The actual movements of the truss points are computed in Table 3, which is based on Principles 1, 3, and 4, stated in Part I. Either Eqs. 6b, 10, 11, and 12a or Eqs. 6a, 10, 11, and 12b may be used. The coordinates for a point in the given truss with respect to the actual fixed point are x_{FP} and y_{FP} recorded in

Cols. 4, Table 3. These coordinates can be written directly by consulting the truss diagram, and Cols. 2 and 3 can be omitted, although they may be used for convenience, or as a check. The projections of the length of a member should have proper signs with respect to its starting point. For example, x_{ch} starts from point c and the truss diagram shows that point h is on the left side of point c; hence, x_{ch} is negative. In Table 3(*b*) this value x_{ch}, given in item 4, Col. 3, plus the value x_{ac} in item 3, Col. 4, yields the value x_{ah} in item 4, Col. 4.

Col. 5, Table 3(*a*), is for $-\theta y_{FP}$ whereas Col. 5, Table 3(*b*), is for $+\theta x_{FP}$. For convenience, the θ-values are entered in items 1 and 6, Col. 5. These spaces are available as it is not necessary to enter the values x''_{FP} and y''_{FP} for the actual fixed points as they are always equal to zero.

TABLE 3.—COMPUTATION OF ACTUAL MOVEMENTS OF TRUSS POINTS

Column No.	Eq. No.	Table (3*a*)	Table 3(*b*)
4	10	$y_{FP} = y_{FQ} + y_{QP}$	$x_{FP} = x_{FQ} + x_{QP}$
5	11	$x''_{FP} = \overrightarrow{F'P''}_x = -\theta y_{FP}$	$y''_{FP} = \overrightarrow{F'P''}_y = +\theta x_{FP}$
6	6*b*	$x''_P = x''_{FP} + x'_F$	$y''_P = y''_{FP} + y'_F$
6 (alternate)		x'_P	y'_P
7[a]		x'_P	y'_P
7 (alternate)	6*a*	$x'_{FP} = x'_P - x'_F$	$y'_{FP} = y'_P - y'_F$
8	12*a*	$\overrightarrow{P''P'}_x = x'_P - x''_P$	$\overrightarrow{P''P'}_y = y'_P - y''_P$
8 (alternate)	12*b*	$\overrightarrow{P''P'}_x = x'_{FP} - x''_{FP}$	$\overrightarrow{P''P'}_y = y'_{FP} - y''_{FP}$

Note: The alternate headings should be taken together.

Item	Joint (Fig. 9)	Member	(*a*) *x*-COMPONENTS					(*b*) *y*-COMPONENTS				
			y_{QP}	y_{FP} (Eq. 10)	x''_{FP} (Eq. 11)	x'_P $(=x'_{FP})$[a]	$\overrightarrow{P''P'}_x$ (Eq. 12*b*)	x_{QP}	y_{FP} (Eq. 10)	y''_{FP} (Eq. 11)	y'_P $(=y'_{FP})$[a]	$\overrightarrow{P''P'}_y$ (Eq. 12*b*)
	(1)	(2)	(3)	(4)	(5)	(6)	(8)	(3)	(4)	(5)	(6)	(8)
1	a$_F$			0	(θ_1=4.511)		0		0	(θ_1=4.511)		0
2	b	ab	+16	+16	−72.18	+100.57	+172.75	+12	+12	+54.13	−25.43	−79.56
3	c	bc	+ 6	+22	−99.24	+55.04	+154.28	+12	+24	+108.26	+230.64	+122.38
4	h	ch	+ 6	+28	−126.31	−42.00	+84.31	−12	+12	+54.13	−68.43	−122.56
5	i	hi	0	+28	−126.31	0	+126.31	−12	0	0	−42.00	−42.00
6	e$_F$				(θ_2=15.675)		0		0	(θ_2=15.675)		0
7	d	ed	+12	+12	−188.10	−33.84	+154.26	...	−16	−250.80	−128.44	+122.36
8	g	dg	+ 6	+18	−282.15	0	+282.15	...	0	0	−24.00	−24.00
9	f	gf	0	+18	−282.15	+64.00	+346.15	...	+16	+250.80	−56.89	−307.69

[a] Col. 7, in the standard form, is not needed in the solution of the present example as $x'_P = x'_{FP}$ and $y'_P = y'_{FP}$ and is omitted.

For Cols. 6, 7, and 8, Table 3, either the set of headings shown or the alternate heads at the top of the table should be used. The values x'_P and y'_P are copied from Table 1. For the given example, either Col. 6 or Col. 7 may be omitted. However, if the origin of the Williot diagram is not the actual fixed point, Cols. 6 and 7 are both necessary. The values in Col. 8 give the components of the actual movements of truss points. (The values given for this example are coefficients of $K = L/(A\,E)$ which is assumed to be the same for all members.)

Some Special Cases

The foregoing example illustrates all the general principles involved in determining truss deflections by the coordinate method. The procedure can be applied to any special case. However, for some cases, the algebraic solution of the Mohr diagram can be much simplified. In certain cases (such as a symmetrical truss under symmetrical loading with the line of symmetry as the fixed member, or a truss with supporting conditions as shown in Fig. 3 using ab instead of ij as the fixed member), no Mohr diagram correction is needed and the coordinates of the points on the Williot diagram will give directly the x-component and the y-component of the true movements of the joints.

1. Consider the case of a truss which satisfies the following conditions: (a) The left end L_o (Fig. 6) is fixed; (b) the right end L_n is movable only in the horizontal direction; (c) the lower chord L_o L_n is horizontal; (d) the upper chord

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
y'_{Lo}	y'_{Ln}	$y'_{Ln}-y'_{Lo}$	$\theta''=\left(\frac{y'_{Ln}-y'_{Lo}}{n}\right)$	m_{eo}	$m_{eo}\theta''$	x'_{Lo}	$x''_{Ui}=x'_{Lo}-m_{eo}\theta''$

(a) DETERMINATION OF θ

(7)	(6)	(5)	(4)	(3)	(2)	(1)(1)'	(2)'	(3)'	(4)'	(5)'	(6)'	(7)'
x'_{Li}	$x'_{Li}-x''_{Li}=(7)-x'_{Lo}$	POINT	$y'_{Li}-y''_{Li}=(3)-(2)$	y'_{Li}	$y''_{Li}=y'_{Lo}+i\theta''$	$i\theta''$	$y''_{Ui}=y''_{Li}=(2)$	y'_{Ui}	$y'_{Ui}-y''_{Ui}=(3)'-(2)'$	POINT	$x'_{Ui}-x''_{Ui}=(7)'-x''_{Ui}$	x'_{Ui}
		L_1				θ''				U_1		
		L_2				$2\theta''$				U_2		
		⋮				⋮				⋮		
		L_i				$i\theta''$				U_i		
		⋮				⋮				⋮		

(b) DETERMINATION OF JOINT MOVEMENT

Fig. 10.—Forms for Computing the Movement of Joints for Ordinary Parallel Chord Trusses

is also horizontal; and (e) the panels are of equal length. After obtaining the values x'_{Li}, y'_{Li}, x'_{Ui}, and y'_{Ui} by filling out a computation form similar to Table 1, the actual movements of the joints can be computed by tabular forms demonstrated by Fig. 10. The value θ'' (Fig. 10(a)) is obtained by applying Eq. 15a and the value x''_{Ui} is obtained by applying Eq. 16; that is, $x''_{Ui} = x'_{Lo} - m_{eo}\,\theta''$ (in which m_{eo} is the slope of the end post at point L_o).

The actual movements of the points are computed by application of Fig. 10(b). The values in Col. 1 are equal to $i\,\theta''$. For example, the value for point L_2 is $2\,\theta''$. The values in Col. 2 are given by the values in Col. 1 plus

y'_{L_o} as shown by Eq. 15*c*. The value of y''_{U_i} in Col. 2′, Fig. 10, is equal to the (y''_{L_i})-value in Col. 2 as shown by Eq. 14*d*. The values x'_{L_i}, y'_{L_i}, x'_{U_i}, and y'_{U_i} in Cols. 7, 3, 7′, and 3′, respectively, are those obtained by the algebraic solution of the Williot diagram. The value of x''_{L_i} is equal to x'_{L_o} according to Eq. 14*a* and it is entered at the proper place in the heading of Col. 6. The value of x''_{U_i} which has been obtained in Fig. 10(*a*) is entered into the heading of Col. 6′. The actual displacements are given Cols. 6, 4, 6′, and 4′, Fig. 10, which are obtained by applying Eq. 17.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	POINT	$x'_{Li}-x''_{Li}$ =(3)-(4)	x'_{Li}	x''_{Li} $=x'_{Lo}$	$\overline{L_hL_i}$	$\theta'\overline{L_hL_i}=$ $(5)\times\theta'$	$y''_{Li}=$ $y''_{Lh}+(6)$	y'_{Li}	$y'_{Li}-y''_{Li}$ =(8)-(7)	POINT
0	L_o		x'_{Lo}	x'_{Lo}		$\theta'=$	$y''_{Lo}=y'_{Lo}$			L_o
1	L_1		x'_{L1}	x'_{Lo}			y''_{L1}	y'_{L1}		L_1
2	•			x'_{Lo}						•
•	•			x'_{Lo}						•
•	•			x'_{Lo}						•
•	•			x'_{Lo}						•
•	•			x'_{Lo}						•
h=i-1	L_h						y''_{Lh}			L_h
i	L_i					$\theta'\overline{L_hL_i}$	y''_{Li}			L_i
j=i+1										

(*a*) LOWER CHORD

	(1)′	(2)′	(3)′	(4)′	(5)′	(6)′	(7)′	(8)′	(9)′	(10)′
	POINT	$x'_{Ui}-x''_{Ui}$ =(3)′-(4)′	x'_{Ui}	$x''_{Ui}=$ $x''_{Uh}+(5)'$	$-\theta'(\overline{U_hU_i})_y$ or $-\theta'\overline{L_hL_i}m_{ci}$	$(\overline{U_hU_i})_y$ or m_{ci}	y''_{Ui} $=y''_{Li}$	y'_{Ui}	$y'_{Ui}-y''_{Ui}$ =(8)′-(7)′	POINT
0	$U_o=L_o$			$x''_{Uo}=x'_{Lo}$	$\theta'=$					
1	U_1									U_1
•	•									•
•	•									•
•	•									•
h=i-1	U_h			x''_{Uh}						U_h
i	U_i			x''_{Ui}	$-\theta'(\overline{U_hU_i})_y$					U_i

(*b*) UPPER CHORD

FIG. 11.—FORMS FOR COMPUTING THE MOVEMENT OF JOINTS FOR CURVED UPPER CHORD TRUSSES

2. If the given truss satisfies conditions (*a*), (*b*), and (*c*) but not conditions (*d*) and (*e*), then the solution can be worked out as follows:

Compute the value of θ' by Eq. 13—either directly or by adapting a table with four columns similar to the first four columns in Fig. 10(*a*). Then compute the actual movements of the joints by the tabular forms shown in Fig. 11. Fig. 11(*a*) applies to the lower chord points and Fig. 11(*b*) to the upper chord points. These tables are based on Eqs. 14 and 17. For two consecutive points L_h and L_i, the value of y''_{L_h} in item h, Col. 7, plus the value $+\ \theta'\ \overline{L_hL_i}$ in line i, Col. 6, gives the value y''_{L_i} in line i, Col. 7. Similarly, x''_{U_h} in line h,

Col. 4′, adding $-\theta'\,(\overline{U_hU_i})_y$ in line i, Col. 5′, gives the value x''_{U_i} in line i, Col. 4′. The values in Col. 5′ can be found either by multiplying $-\theta'$ by the y-projection of the chord U_hU_i (as shown in the truss diagram and as recorded in Col. 6′), or by multiplying the value $\theta'\,\overline{L_hL_i}$ in Col. 6 by $-m_{ci}$ given in Col. 6′. The value m_{ci} is the slope of U_hU_i, and $(\overline{U_hU_i})_y$ is its y-projection, which should have the same sign as m_{ci} when point U_h is the left end of member U_hU_i. The values in Col. 7′ are the same as those in Col. 7. The coordinates of L'_i and U'_i given in Cols. 3, 8, 3′, and 8′ are those found by the algebraic solution of the Williot diagram. The components of the actual movements of truss joints are given in the Cols. 2, 9, 2′, and 9′.

Conclusion

The procedure described herein is an algebraic equivalent of the graphical method known as the Williot-Mohr diagram. The sign conventions suggested are easy to remember. The formulas to be used are rather simple in form; and they can be transformed into simple operations arranged in tabular forms. For a rather complicated problem, not more than three forms of tables need be used. When the designer is familiar with the operations, he can forget all the formulas.

The knowledge of truss deflections is important because it has applications in many cases such as cambering, erection by the cantilever method, stress analysis for indeterminate trusses, and the computation of secondary stresses.

Acknowledgment.—The writer wishes to express thanks to T. C. Shedd, M. ASCE, of the University of Illinois at Urbana for his encouragement in the course of study.